An Investigation of the Impacts of Phase Noise on Symbol Error Rate in Quadrature Amplitude Modulation Systems

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Abstract— The existence of phase noise in virtually every digital communications system poses a serious challenge to system designers especially as system complexity increases. Communication system complexity could be attributed to the modulation techniques adopted and the circuitry employed in achieving such modulations. This work investigates the impact of phase noise on the Symbol Error Rate (SER) of the different Quadrature Amplitude Modulation (QAM) schemes. MATLAB simulation technique is adopted for the work and the results of the simulations show that as the phase noise is increased negatively, all the QAM schemes investigated show a reduction in SER and at a point, records a zero error. The value at which the schemes record this zero symbol error rate increases as the complexity of the scheme increases. So, higher order QAM schemes accommodate more symbol errors than the lower orders. The results also show that hard decision decoding has the worst performance index, irrespective of the QAM scheme, when compared with soft decision decoding.

Keywords— Bit error rate, Phase noise, Quadrature Amplitude Modulation, Symbol error rate,.

1. INTRODUCTION

The tremendous growth and advancement recorded in telecommunication technology in recent times have been as a result of increasing demand for capacity and diversity, the zeal to overcome the limitations of older generations of technology, as well as the hunger for improved user experience. This exponential growth in bandwidth demand necessitates the growing trend towards having communications systems with high capacity (Seimetz, 2009). Some of the metrics for determining how good a telecommunications system is, include the system capacity, signal coverage, signal reproducing capability, and bit error rate (BER).

Impairment factors, however, pose a lot of challenges to the efficiency and effectiveness of these systems. The third and fourth generation (3G and 4G) of wireless networks demand communication links that are not only of high quality but are also capable of handling high data rate (Lari et al, 2013). One good choice that is suitable in this regards is the quadrature amplitude modulation (QAM) technique. The suitability of QAM is however punctured by its vulnerability to some impairment factors such as phase noise, DC offset, nonlinearity of amplifier, phase and amplitude imbalance. This paper focuses on the phase noise and investigates the sensitivity of M-ary QAM to variations in phase noise level.

2. LITERATURE REVIEW

2.1 Theoretical Background

Sine wave is defined by three characteristics: amplitude, frequency, and phase. Varying any of these characteristics changes the electrical property of the signal and as such, can be used to represent digital data. When the amplitude is altered, it is called amplitude shift keying (ASK).

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Altering of frequency results is frequency shift keying (FSK), while that of phase is known as the phase shift keying (PSK). All these are used in modulating digital data unto an analog signal (Forouzan, 2006). In ASK, the two binary values are represented by two different amplitudes of the carrier frequency. Here, the resulting signal is given as (Stallings, 2007):

$$s(t) = \begin{cases} V\cos(2\pi f_c t) & binary\ 1 \\ 0 & binary\ 0 \end{cases} \qquad (1)$$

Despite its wide application range, ASK is adversely affected by sudden gain changes and as such is inefficient (Stallings, 2007). In FSK, two different frequencies within the neighborhood of the carrier frequency are used in representing two binary values. The resulting signal is (Stallings, 2007):

$$s(t) = \begin{cases} V\cos(2\pi f_1 t) & binary\ 1 \\ V\cos(2\pi f_2 t) & binary\ 0 \end{cases} \qquad (2)$$

In PSK, the data is represented by shifting the phase of the carrier signal. This results in a signal (Stallings, 2007):

$$s(t) = \begin{cases} V\cos(2\pi f_c t + \pi) & binary\ 1 \\ V\cos(2\pi f_c t) & binary\ 0 \end{cases} \qquad (3)$$

A hybrid mechanism which has been adjudged as the most efficient, combines both the amplitude and phase and is known as quadrature amplitude modulation (QAM). QAM is a 2-dimensional signaling scheme in which the original information stream is split into two sequences consisting of odd and even symbols. So there is a pair of quadrature carriers, normally a sine and a cosine waveform, which can be modulated independently and subsequently transmitted using the same frequency band (Hanzo et al, 2004). Suppose the two sequences are A_k and B_k as shown in figure 1, where A_k sequence is the in-phase component which is usually modulated by $\cos(2\pi f_c t)$ while B_k sequence is the quadrature-phase component which is usually modulated by $\sin(2\pi f_c t)$.

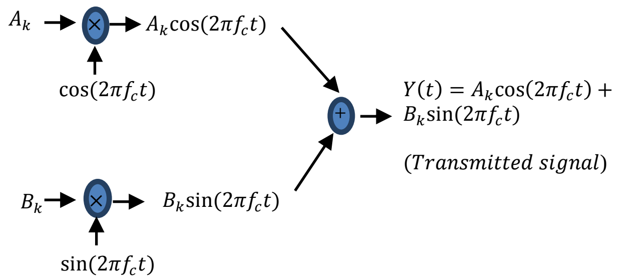

Fig. 1: Schematic diagram illustrating Quadrature Amplitude Modulation (Hanzo et al, 2004)

QAM has an advantage of double bit data rate. That is, the data rate equals two bits per bit interval. Consider a_k and b_k as a coordinate pair of data-symbol to be transmitted. The signals at the receiver are given by (Crawford, 2004):

$$I_k = R[(a_k + jb_k)e^{j\varphi_n}] = R[(a_k + jb_k)(cos(\varphi_n) + jsin(\varphi_n))] = a_k cos(\varphi_n) - b_k sin(\varphi_n) + n_I \quad (4)$$

$$Q_k = Im[(a_k + jb_k)e^{j\varphi_n}] = Im[(a_k + jb_k)(cos(\varphi_n) + jsin(\varphi_n))] = a_k sin(\varphi_n) + b_k cos(\varphi_n) + n_Q \quad (5)$$

Where φ_n is the phase noise while n_I and n_Q are I- and Q- additive Gaussian noise terms. The only term common to both the I and Q channels is the phase noise φ_n. This makes the I and Q channels to be cross-coupled with the attendant cross-talk problems.

During detection, a symbol error occurs if there is a detection decision error on either the I- or Q-channel. This occurs when the absolute value of the difference between the received signal and transmitted signal on either channel is greater than the detection threshold (Crawford, 2004). That is, Symbol error occurs when:

$$|(a_k cos(\varphi_n) - b_k sin(\varphi_n) + n_I) - a_k| > D_t \quad (6)$$

Or:

$$|(a_k sin(\varphi_n) + b_k cos(\varphi_n) + n_Q) - b_k| > D_t \quad (7)$$

Where D_t is a function of the voltage distance between signal levels. The probability of receiving a given symbol without error is given by (Crawford, 2004):

$$P_{sym}(\varphi_n) = 2\left(\frac{M-1}{M}\right)\left\langle erfc\left[\frac{\frac{d}{2} - a_k sin(\varphi_n)}{\sigma\sqrt{2}}\right]\right\rangle \quad (8)$$

Where angular brackets denote averaging over the a_k's, M is the signal level, and d is the (voltage) distance between signal levels.

On the other hand, the probability of detecting a symbol in error in the presence of phase noise is given by (Crawford, 2004):

$$P_{SymErr} = \int_{-\infty}^{+\infty} P_{sym}(\varphi_n) P_{\theta_n}(\varphi_n) d\varphi_n \quad (9)$$

When there is no phase noise, the probability of detecting a symbol in error is given by (Proakis, 1989):

$$P_{sym}(\gamma_b, M, k) = 2\left(\frac{M-1}{M}\right) erfc\left[\sqrt{\frac{3k\gamma_b}{2(M^2-1)}}\right] \cdot \left[1 - \frac{1}{2}\left(\frac{M-1}{M}\right) erfc\left[\sqrt{\frac{3k\gamma_b}{2(M^2-1)}}\right]\right] \quad (10)$$

Where γ_b is the SNR per bit, k is the number of bits per symbol while M is the number of levels on each rail of a square-QAM (for instance, for 64-QAM, $M = 8$)

2.2 Phase Noise

One of the distinguishing factors between real and ideal oscillators is that the later generates a pure sine wave while the former has phase modulated noise components. The resulting signal from ideal oscillators has the following signal representation: $s(t) = V\sin(2\pi ft)$ while the real oscillators are represented as: $s(t) = (V + v(t))\sin(2\pi ft + \varphi(t))$, where $v(t)$ is the amplitude noise while $\varphi(t)$ is the phase noise. Digital modulations are therefore sensitive to impairments that disturb the phase such as the phase jitter (Howald, 2001).

Phase noise is a random fluctuation in the phase of an oscillator waveform (Lari et al, 2013). The major causes of phase noise are the various components and circuits used within a signal generator or oscillator. These factors disperse output power to the surrounding frequencies (Agilent, 2011). Hence according to (Esterline, 2008), Phase noise is an undesired entity present in all real world signal generators and oscillators which causes distortion of incoming information in signal receivers.
Phase noise is widely used in describing short term random frequency fluctuations of a signal. The measure of the degree to which a signal generator maintains the same frequency over a period of time is known as frequency stability. This frequency stability could be described using any of the three signal characteristics already discussed above namely; amplitude, frequency and phase. Phase can either be discrete or random. When it is the random phase fluctuation, it is referred to as phase noise.

2.3 Related Work

Minimizing phase noise remains essential in order for any communications system to achieve its greatest potential (Vye, 2004). The primary trade-off in integrated oscillator design is between an acceptable low phase noise and power dissipation (Ahrens and Thomas, 1998). This implies that as phase noise decreases, the power dissipation increases. Phase noise degrades in proportion to frequency squared (Hajimiri and Lee, 1998). In (Crawford, 2004), Phase noise is shown as an increasingly serious performance issue as the order of the QAM signal constellation is increased although, only square-QAM signal constellations are considered.

Khanzadi et al, (2014) investigates the effect of oscillator phase noise and channel variations due to fading on the performance of communication systems at frequency bands higher than 10GHz. Their results show that phase noise can have severe effects on the system performance at high frequencies. Results here also show that performance degradation due to phase noise can be more severe when the center frequency is increased and the bandwidth is kept a constant, or when oscillators based on low power CMOS technology are used. In (Menon et al, 2015), a modelling and simulation of combined phase and amplitude noise for QAM direct conversion receivers was carried out. The result shows that for small phase noise, the SER approaches theoretical SER for only amplitude noise case and increases to large values for large phase noise.

The effect of phase noise for various laser linewidths on system performance is given in (Hussin, Noe and Panhwar, 2014). Here, an investigation of a common phase error equalizer for coherent optical OFDMA with radio frequency pilot phase noise compensation is carried out. Khanzadi et al., (2014) mathematically derived and analyzed a direct connection between oscillator measurements and the optimal performance of communication system. They found out that the influence from different noise regions strongly depends on the symbol rate. For high symbol rate communication systems, cumulative PN that appears near carrier is of relatively low importance compared to the white PN far from carrier. Their results also show that $1/f^3$ noise has a lower effect on communications system performances than the $1/f^2$ noise.

Due to the undesirability of phase noise, a lot of work has been done in a bid to reduce its effect on communication systems. In (Pakala and Schmauss, 2014) an optimal carrier phase and amplitude noise estimation (CPANE) algorithm using extended Kalman filter (EKF) is proposed for effective mitigation of linear and nonlinear phase noise with simultaneous suppression of amplified spontaneous emission (ASE) noise. Umeki et al, (2014) shows a phase sensitive amplification of a high-order quadrature amplitude modulation (QAM) signal using non-degenerate parametric amplification in a periodically poled lithium niobate (PPLN) waveguide. The study shows that phase sensitive amplifier has the capability of reducing both the phase noise and amplitude. Jain et al, (2017) shows that phase noise, frequency offset, chromatic dispersion, and fiber nonlinearities affect the performance of higher order QAM coherent optical communication systems. An extended Kalman filter (EKF) algorithm is proposed to jointly mitigate laser phase noise, frequency offset, and nonlinear channel impairments. The problem of self-interference cancellation with phase noise suppression in full-duplex systems is considered in (Ahmed and Eltawil, 2015). The authors also proposed two different phase noise suppression techniques. A novel closed form expressions for the error vector magnitude (EVM) which combines the in-phase quadrature (IQ) amplitude and phase imbalances and the DC offsets along with the phase noise, is presented in (Georgiadis and Kalialakis, 2014). Here, both the Tikhonov and the Gaussian probability density functions are utilized for the oscillator phase noise distribution and point of convergence of the two distributions is investigated.
The aim of this work is to investigate the sensitivity of phase noise on M-ary Quadrature Amplitude Modulation using computer aided design tool.

3. METHODOLOGY

The investigative research carried out in this work was done using computer simulation. Precisely, MATLAB Simulation models were used. Two MATLAB models, figure 2 and figure 3, were used. Figure 2 shows a model for investigating the effects of phase noise in QAM systems. The model is composed of the following MATLAB model blocks: Random integer generation block, rectangular QAM modulator baseband, rectangular QAM demodulator baseband, Additive White Gaussian Noise (AWGN) channel, Phase noise block, and error calculator. The block parameters were varied and results of the simulations recorded. For the pair of modulator and demodulator, the M-ary numbers were varied for M equals 4, 16, 32, 64, 128, and 256. For the phase noise block, the phase noise level was also varied to see the effects on the QAM system while a frequency offset value of 200Hz was used. In the Additive White Gaussian Noise block, the mode was set to Eb/No.

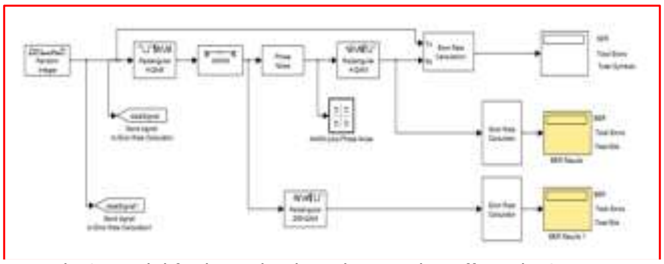

Fig 2: Model for investigating Phase noise effects in QAM

Figure 3 shows a model for verifying the effects of phase noise on bit error rate. This model is composed of the following MATLAB blocks: Bernoulli binary generator, convolutional encoder, rectangular QAM modulator baseband, AWGN block, phase noise, error rate calculator, noise variance block with constant value output, and decoding subsystem. Figure 3 is a MATLAB model for investigating the effects of phase noise on BER using different decoding techniques. The techniques used are Hard decision decoding, Log-Likelihood Ratio (LLR) + Soft decision decoding, and LLR + unquantized decoding. The LLR metric helps in improving the performance of the error correction. The decoding subsystem houses the model blocks for the three different decoding techniques.

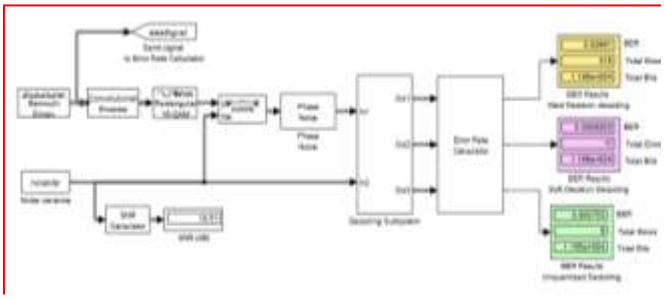

Fig 3: Model for verifying effects of phase noise on BER

4. Simulation Results and Discussions

Figure 4 shows the effect of phase noise introduction on symbol error rate. Lower values of M have lower SER when there is no phase noise. At -6dBc/Hz phase noise, the lower M-ary QAM schemes also have lower SER but the difference between the SER for 4-QAM and 256-QAM is smaller than what is observed when there is no phase noise. As the phase noise is increased to -46dBc/Hz, there is a linear reduction in the value of the SER and there is a divergence trend in the values as the phase noise increases.

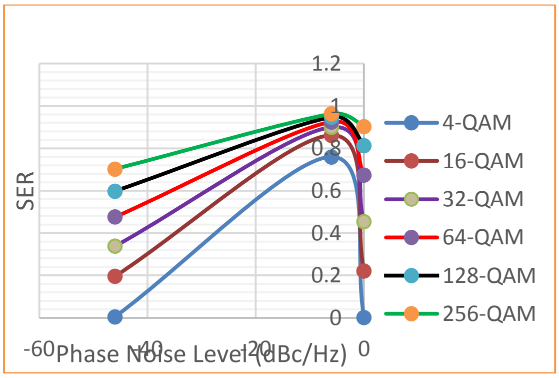

Fig 4: Effect of introducing Phase Noise on SER

As observed in figure 4, figure 5 shows that there is a downward trend in the values of the SER as the phase noise is increased (in the negative direction). Then at around -50dBc/Hz, the 4-QAM records a zero SER, which implies that there is zero error at this level. The value of phase noise at which the 256-QAM records a zero SER is -71dBc/Hz

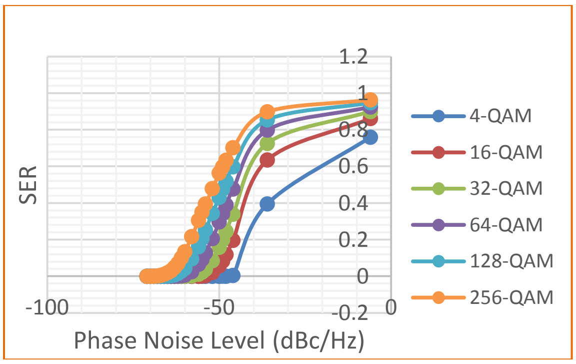

Fig 5: Phase Noise versus Symbol Error Rate

Figure 6 shows that for any particular phase noise level, the total number of symbol errors increases with increase in the value of M in M-ary QAM. The figure also shows that lower orders of QAM have higher slopes unlike the higher orders which implies that the higher orders record a slower change with phase variation particularly between -6dBc/Hz and -36dBc/Hz.

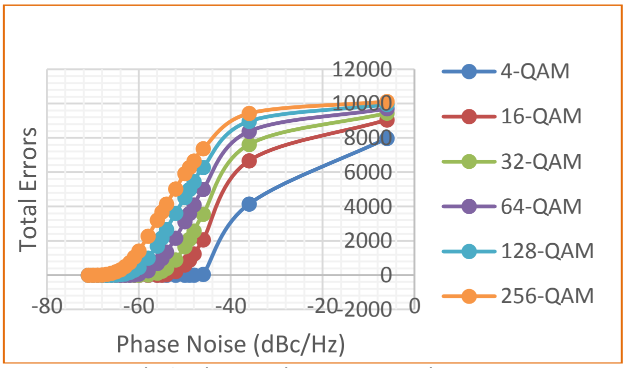

Fig 6: Phase Noise versus Total errors

Figure 7 shows that between 0 and 10 dB of the Eb/No, there is a faster change in SER for the lower orders of QAM as indicated by their steeper slopes. But as the Eb/No increases, these lower orders attain earlier, a stable SER than the higher orders.

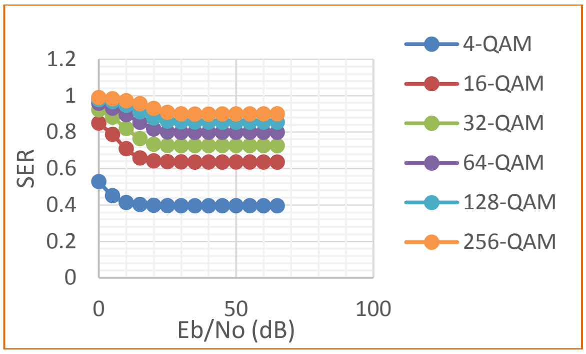

Fig 7: Symbol Error Rate versus Eb/No

Figure 8 and Figure 9 show the effect of varying the decoding techniques on phase noise. Both figure indicate that the hard decision decoding has the worst performance index. Then for lower order 16-QAM, there is a kind of overlap between the performances of soft decision decoding and unquantized decoding but higher order 256-QAM exhibits a different set of performance for the soft decision and unquantized decoding. At -36dBc/Hz, unquantized decoding records a higher bit error rate but at -51dBc/Hz, soft decision decoding records a higher bit error rate.

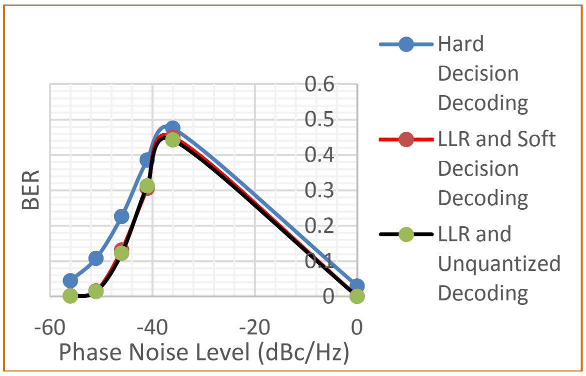

Fig 8: Effect of Phase Noise on BER for 16-QAM

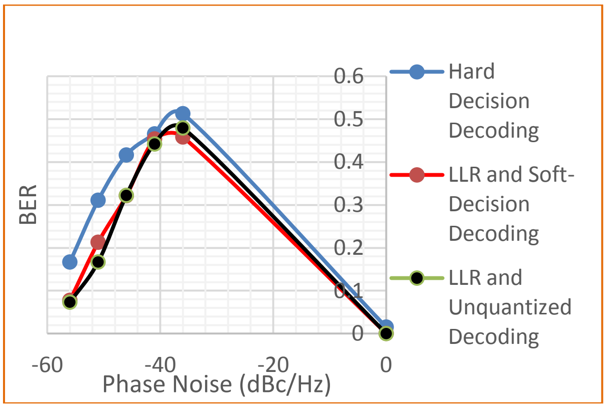

Fig 9: Effect of Phase Noise on BER for 256-QAM

5 Conclusion

As the phase noise is increased negatively, all the QAM schemes investigated show a reduction in SER and at a point, records a zero error. The value at which the schemes record this zero symbol error rate increases as the complexity of the scheme increases. This means that higher order QAM schemes accommodate more symbol errors than the lower orders. Lower orders of QAM have higher 'total error versus phase noise' slopes unlike the higher orders which implies that the higher orders record a slower change with phase variation particularly between -6dBc/Hz and -36dBc/Hz.

Although, irrespective of the QAM scheme, hard decision decoding has the worst performance index, the performances of the soft decision decoding and unquantized decoding overlap for the lower order QAM but have different sets of bit error rates for higher order QAM. The direct implication is that hard decision decoding is not energy efficient since it records high BER irrespective of the order of the QAM. Consequently, when achieving energy efficiency is an objective in the design of QAM systems, hard decision decoding is not an option.

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